Math for CS. Session 24. Take-Home Problem Set

1) Let S_{99} be the sum of all numbers on the 99-th line of the Pascal triangle, and S_{100} be the sum of all numbers on the 100-th line. Compute S_{100}/S_{99}.

2) Robot Rolly inserted alternating "+" and "–" signs in the 99th row of Pascal's triangle. Next, she calculated the resulting expression. What was her answer?

3) Prove that in a Pascal triangle the central number in a row is equal to the sum of all the numbers in a diamond-shapes square above it (see pic)
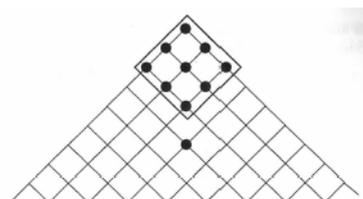
4) Mario needs to get from the lower-left corner to the upper-right corner of the minefield in 7 steps without getting himself blown up on the mines. In how many ways can he do that?
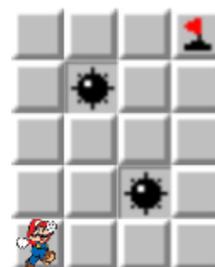
5) Prove that $1 * 2 + 2 * 3 + \cdots + (n - 1) * n = (n - 1)n(n + 1)/3$

6) Prove that

$$\frac{1}{1 * 2} + \frac{1}{2 * 3} + \cdots + \frac{1}{(n - 1) * n} = \frac{n - 1}{1}$$

7) In "Minesweeper", some squares of a 10x10 board contain mines. The computer fills other squares with the number of neighboring squares that have mines.
Let us suppose I erase all mines and then put new mines on all squares that previously did not have a mine. Can the sum of all numbers on the board decrease, increase, stay the same?
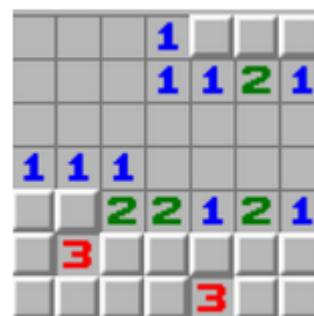
8) You have 100 pancakes, one with a single blueberry, one with two blueberries, one with three blueberries, and so on. The pancakes are stacked in a random order. Count the number of blueberries in the top pancake, and call that number N. Pick up the stack of the top N pancakes, and flip it upside down. Prove that if you repeat this counting-and-flipping process, the pancake with one blueberry will eventually end up at the top of the stack. (UW Math Hour Olympiad 2016)