

Prime Factor Math Circle 2016-2017

Intermediate Circle 2, Problem Set 26

May 15th, 2017

1. An evil wizard is trying to create a magic spell that will destroy the world. She starts by filling a 10×13 table (10 rows and 13 columns) with numbers in such a way that the product of the numbers in each row is negative. She will then use this table to conjure her spell.

The good wizard wants to cast a counter spell (and to save the world). To do so, she must find a column in the evil wizard's table such that the product of all numbers in the column is positive. Prove that the good wizard is always able to save the world.



2. How many ways are there to build a 1×9 wall using 1×1 blue bricks, 1×2 red bricks, and 1×3 green bricks?

3. (MHO 2014, grade 6-7, 8-10)

Hermione and Ron play a game that starts with 129 hats arranged in a circle. They take turns magically transforming the hats into animals. On each turn, a player picks a hat and chooses whether to change it into a badger or into a raven.



A player loses if after his or her turn there are two animals of the same species right next to each other. Hermione goes first. Who loses?



4. Three pirates found 7 diamonds and plan to share them between themselves. How many ways are there for them to distribute these diamonds, if:

- a) Each of them must get at least 1 diamond.
- b) It is not required that everyone gets at least one diamond.



5. Max and Thomas have two piles of cards, 20 cards in one pile, and 21 cards in the other. On each turn, a player takes all cards from one of the piles, and splits the other pile into two (not necessarily equal) piles. The player who cannot make a move loses. (For example, if there is two piles 1 card each, the player cannot make move.)

Max goes first. Who has a winning strategy?