

Prime Factor Math Circle 2016-2017

Math for CS. Problem Set 26. Take-Home Problem Set

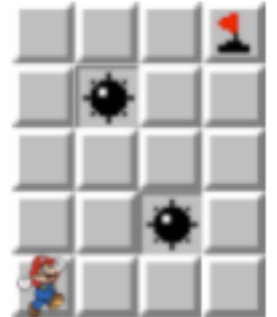
Reminder: $C\binom{n}{k}$, which is also denoted $\binom{n}{k}$ tells in how many way we can chose k objects out of n if the order we choose the objects does not matter.

$\binom{n}{k}$ to be read as “ n choose k ”)

$$\binom{n}{k} = \frac{n!}{k! \times (n-k)!}$$

- 1) Prove that $\binom{n}{n-k} = \binom{n}{k}$
- 2) Prove that $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$
- 3) In how many ways can you choose 4 colors out of 7 given colors?
- 4) There are three rooms in the dormitory: one single, one double, and one for four students. How many ways are there to house seven students in these rooms?
- 5) 40 ladies and 40 gentlemen came for the Fall Ball. IN how many ways can they split into pairs for the first dance?

- 6) Mario needs to get from the lower-left corner to the upper-right corner of the minefield in 7 steps without getting himself blown up on the mines. In how many ways can he do that?



- 7) What is the fewest number of terms you need to cross out from $99!$ to get a product that ends on 2?
- 8) In a product of 3 integer numbers, each term was replaced by a number that is 3 smaller. Could the product become 2016 bigger? (Either prove that this is not possible, or come up with an example.)
- 9) In the product $1! \times 2! \times 3! \times \dots \times 100!$ cross out one of the factorials so that the remaining number becomes a perfect square.