

# Prime Factor Math Circle 2016-17

## Intermediate Circle 2, Problem Set 9

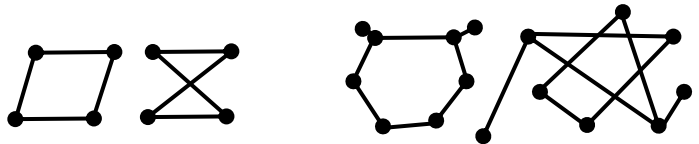
Nov. 28, 2016

1. A certain kingdom consists of 21 provinces. Can it happen that each province has either 1, or 5 or 9 neighbors?



2. Two graphs are called isomorphic if they have the same number of vertices and edges, and their vertices could be mapped into each other in such a way that two vertices in one graph are connected if and only if they are connected in the second graph. In other words, isomorphic graphs represent the same collection of vertices and edges, only drawn in different ways.

Examples of isomorphic graphs:

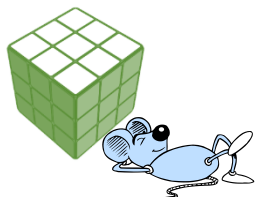


Is it true, that two graphs are isomorphic, if:

- Each has 8 vertices and the degree of each vertex is 7?
  - Each has 8 vertices and the degree of each vertex is 3?
3. In a certain county, 100 roads lead out of each city, and one can travel along those roads from any city to any other. One road is closed for repairs. Prove that one can still get from any one city to any other.



4. Lily the Mouse found a  $3 \times 3$  piece of cheese made of 27 small cheese cubes. Lily can eat one small cube a day; after she eats a cube she falls asleep right away. She gets up the next day and eats another small cube that shared a face with the one she ate the day before. Could it happen that after 26 days of this cheese eating spree only the central cheese cube is left?



5. Prove that out of any 6 people, there are either 3 who all do not know each other, or 3 who all do know each other.

